

Workplace and Apprenticeship 10

Unit 5 – Angles and Parallel Lines

WA 10.9 Demonstrate understanding of angles including:

- drawing and sketching
- replicating and constructing
- bisecting
- relating to parallel, perpendicular, and transversal lines
- solving problems.

** Adapted from Pacific Educational Press MathWorks 10*

Angles & Parallel Lines - Key Terms

<p>Point - A precise location or place on a plane. Usually represented by a dot.</p>	<p>Line - A geometrical object that is straight, infinitely long and infinitely thin.</p>
<p>Line Segment - All points between two given points, including the points themselves (the line between two points)</p>	<p>Ray - A portion of a line which starts at a point and goes off in a particular direction to infinity.</p>
<p>Angle – two rays or line segments that meet at a point, called the vertex.</p>	<p>Vertex - The common endpoint of two or more rays or line segments.</p>
<p>Angle Measure – the gap between the two rays, expressed in degrees.</p>	<p>Acute Angle - refers to an angle less than 90°.</p>
<p>Right Angle – an angle whose measure is exactly 90°.</p>	<p>Obtuse Angle - an angle between 90° and 180°.</p>

<p>Straight Angle – An angle whose measure is exactly 180° (a straight line)</p>	<p>Reflex Angle – An angle that is between 180° and 360°.</p>
<p>Angle Referents – A common standard of an angle measure, for example: 0°, 30°, 45°, 60°, 90°, 180°, 360°.</p>	
<p>Complementary Angles - Two angles that have measures that add up to 90°</p>	<p>Supplementary Angles – Two angles that have measures that add up to 180°.</p>
<p>True Bearing – The angle measured clockwise between true north and an intended path or direction, expressed in degrees.</p>	<p>True North – Has a bearing of 0°.</p>
<p>Clockwise - In a circle corresponding in direction to the movement of the hands of a clock. (To the right)</p>	<p>Counter Clockwise - In a circle corresponding in direction to the movement opposite of the hands of a clock. (To the left)</p>

<p>Protractor - A protractor is a tool used to measure angles</p>	<p>Compass - A compass is a drawing instrument used for drawing circles and arcs.</p>
<p>Construction - act of drawing geometric shapes using only a compass and straightedge</p>	<p>Congruent – The same shape and size.</p>
<p>Bisect – A bisector is something that cuts an object into two equal parts</p>	<p>Angle Bisector - A segment, ray, or line that separates two halves of a bisected angle.</p>
<p>Perpendicular Lines - A line is perpendicular to another if it meets or crosses it at right angles (90°).</p>	<p>Parallel Lines - Lines are parallel if they lie in the same plane, and are the same distance apart over their entire length</p>
<p>Non-Parallel Lines – lines that are not parallel and will converge (cross)</p>	<p>Transversal – A line that intersects two or more lines.</p>

<p>Corresponding Angles – Two angles that are congruent and occupy the same relative position in similar figures or at different intersections.</p>	<p>Vertically Opposite Angles – angles created by intersecting lines that share only a vertex.</p>
<p>Alternate Interior Angles – Angles in opposite positions between two lines intersected by a transversal and also on alternate sides of the same transversal.</p>	<p>Alternate Exterior Angles – Angles in opposite positions outside two lines intersected by a transversal.</p>
<p>Same Side Interior Angles – angles in the same position between two lines intersected by a transversal.</p>	<p>Same Side Exterior Angles – angles in the same position on the outside of two lines intersected by a transversal.</p>
<p>Angles & Parallel Lines – Theorems</p>	
<p>If two lines are cut by a transversal, then:</p> <ul style="list-style-type: none"> - Alternate interior angles are equal - Corresponding angles are equal - Interior angles on the same side of the transversal are supplementary 	<p>If two PARALLEL lines are cut by a transversal, then</p> <ul style="list-style-type: none"> - The alternate interior angles are equal - The corresponding angles are equal - The interior angles on the same side of the transversal are equal.

Lesson 1 – Angles

Acute Angle - refers to an angle less than 90° .

Right Angle – an angle whose measure is exactly 90° .

Obtuse Angle - an angle between 90° and 180° .

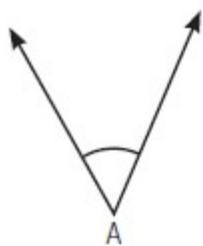
Straight Angle – An angle whose measure is exactly 180° (a straight line)

Reflex Angle – An angle that is between 180° and 360° .

Referent Angles are common angles that are easy to visualize or estimate.

For example: 180° , 90° , 45° , 22.5° , 30° , 60° , 135°

Example 1: Using referent angles, estimate the angles below within 5°



Finding Complementary Angles - Subtract the angle measured from 90° .

Finding Supplementary Angles - Subtract the angle measured from 180° .

Example 2: Given the following angles, determine the size of the a) complement and b) supplement (if they exist).

a) 75°

b) 43°

c) 103°

d) 87°

e) 300°

True Bearing – The angle measured clockwise between true north and an intended path or direction, expressed in degrees.

True North – Has a bearing of 0° .

Calculating True Bearing

- 1) Starting at north, going counter clockwise, draw a circle from the north axis to the direction you are going.
- 2) Determine the number of angles you have gone based on the true bearing compass

Example 3 – A boat is heading directly northwest. What is its true bearing?

Example 4 – You are travelling on a bearing of 225° . What direction are you going?

Worksheet 1

1. Identify the type of angle: acute, right, obtuse, straight, or reflex.

a) 68°

b) 215°

c) 91°

d) 32°

e) 180°

f) 99°

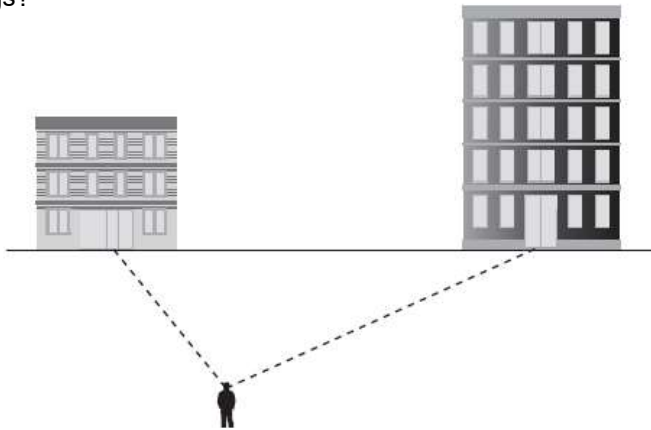
g) 195°

h) 265°

2. Use referents to determine the approximate size of the following angles:



3. Jason is doing a survey of a city block. What is the approximate angle between his sightings of the two buildings?



Fill in the chart with the complement and the supplement of each angle, if they exist.

4. Fill in the chart with the complement and the supplement of each angle, if they exist. If they don't exist, state why.

ANGLE COMPLEMENTS AND SUPPLEMENTS		
Angle	Complement	Supplement
45°		
78°		
112°		
160°		
220°		

5. a) The complement of an angle is 58° , what is the size of the angle?

b) What is the supplement of the angle?

6. a) The complement of an angle is 0° , what is the size of the angle?

b) What is the supplement of the angle?

7. If a boat is traveling 25° south of straight east, what is its true bearing?

8. What is the true bearing of a boat travelling south?

9. What is the true bearing of a boat travelling north-northwest?

10. If Renata cuts a rectangular tile diagonally, one of the acute angles formed is 65° . What is the size of the other acute angle?

Lesson 2 - Constructions

Construction - Act of drawing geometric shapes using only a compass and straightedge.

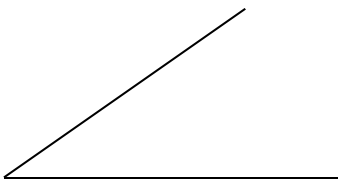
Steps to Construct a Right Angle:

- 1) Draw a line segment
- 2) Mark a point near the middle where you want to make the right angle
- 3) Put the point of the compass on the point
- 4) Open the compass slightly
- 5) Make a small line on each side of the point where it intersects the line segment
- 6) Put the point of the compass on one of the small lines
- 7) Open the compass to the line on the other side
- 8) Draw an arc above the line segment
- 9) Put the point of the compass on the other small line
- 10) Make an arc above the line segment (This should make an "x")
- 11) Draw a straight line from the original point to the "x" above the line segment.

Example 1: Construct a right angle.

Steps to Reconstruct an Existing Angle

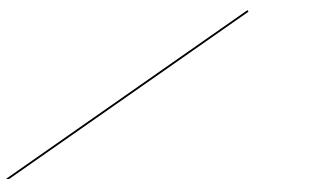
- 1) Draw a ray on your page. This will be the bottom ray of your new angle.
- 2) Put the point of the compass on the vertex of the original angle.
- 3) Draw an arc on the original angle
- 4) Draw the same arc on the new angle
- 5) Put the point of the compass on the original angle where the arc intersects the bottom and make the width of the compass from that point to where the arc intersects the top of the angle.
- 6) Put the compass on the new angle on the point where it intersects the arc on the bottom
- 7) Make an arc (it should make an 'x')
- 8) Draw a straight line from the end of your new ray through the 'x'



Angle Bisector - A segment, ray, or line that separates two halves of a bisected angle.

Steps to Bisect an Angle

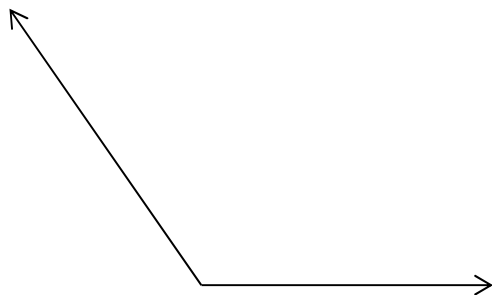
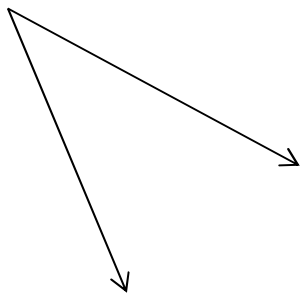
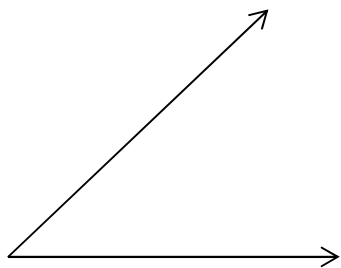
- 1) Put your compass on the vertex of the angle
- 2) Change the width of the compass
- 3) Make an arc that goes through the top and bottom ray of the angle
- 4) Put your point on the point where the arc intersects the top of the angle
- 5) Make the width of the compass from that point, to the point where the arc intersects the bottom of the angle
- 6) Make an arc at that width on the outside of the angle
- 7) Without changing the width, put the compass on the point where the bottom intersects the arc
- 8) Make an arc on the outside of the angle, should make an 'x' with the other arc
- 9) With a straight edge connect the vertex and the 'x'



Worksheet 2

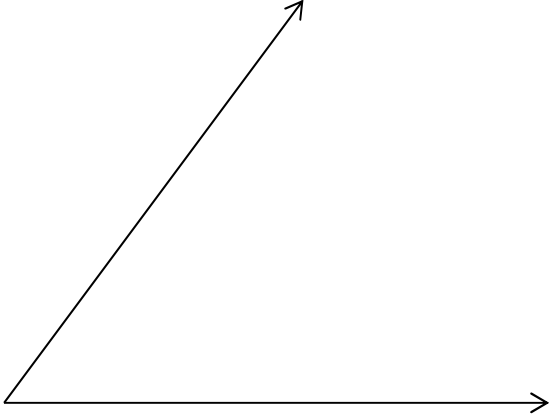
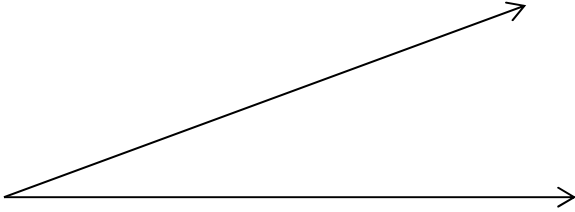
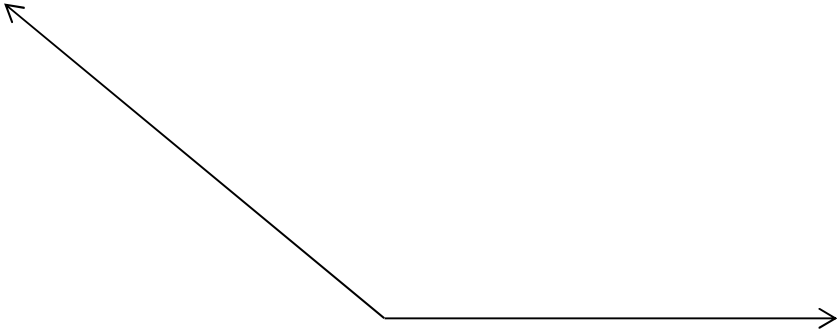
Use only a straight edge and a compass to make two ninety degree angles.

Reconstruct the existing angle in the space provided.



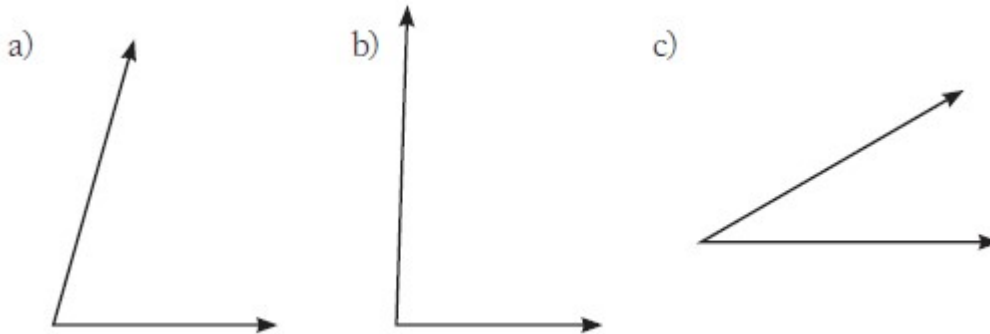
Construct an Angle Bisector

Use a compass and a straight edge to bisect the angles below.

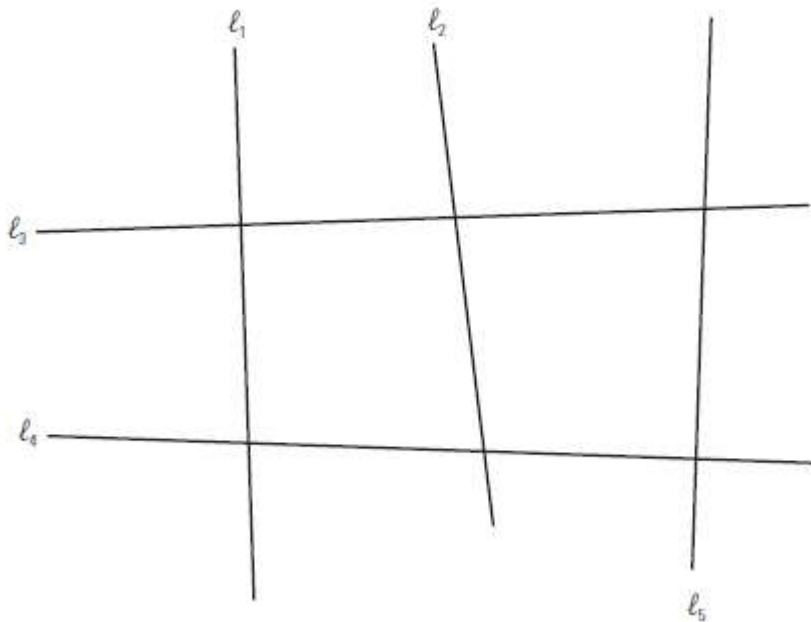
Lesson 3 - Angle Bisectors & Perpendicular Lines

Example: Determine the measure of the resulting angle after the original angle has been bisected.



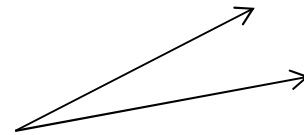
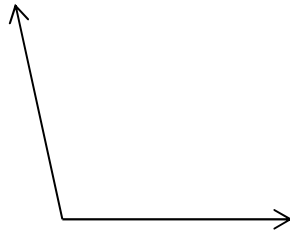
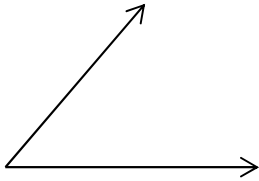
Example: An angle is bisected. Each resulting angle is 136° . How big was the original angle?

Example: Using a protractor, determine which of the following lines are perpendicular.

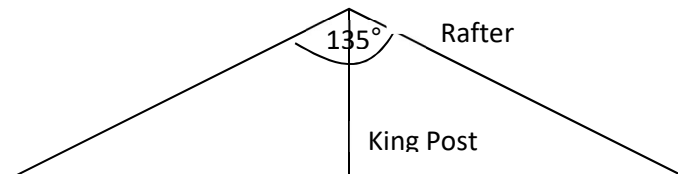


Worksheet 3

1. If a right angle is bisected, what is the size of each angle?
2. Bisect the following angles by measuring the degrees. Label the measure of each bisected angle.



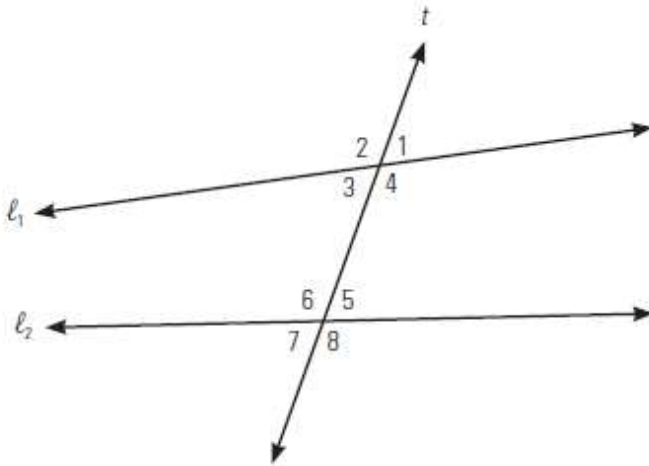
3. An angle is bisected. Each resulting angle is 78° . How big is the angle? What kind of angle is it?
4. The size of an angle is 154° . What is the measure of each bisected angle?
5. A crooked table leg makes an angle of 86.7° with the table top. How much must the carpenter move the leg so that it is perpendicular to the top?
6. The angle at the peak of a roof is 135° . Calculate the measure of the angle formed by the rafter and the king post.



Lesson 4: Non-Parallel Lines and Transversals

A line that intersects two other lines at two distinct points is a **transversal**. When two non-parallel lines are intersected by a transversal, they form angles of varying sizes.

Consider the diagram below: t is a transversal that intersects l_1 and l_2 .



Eight angles are formed.

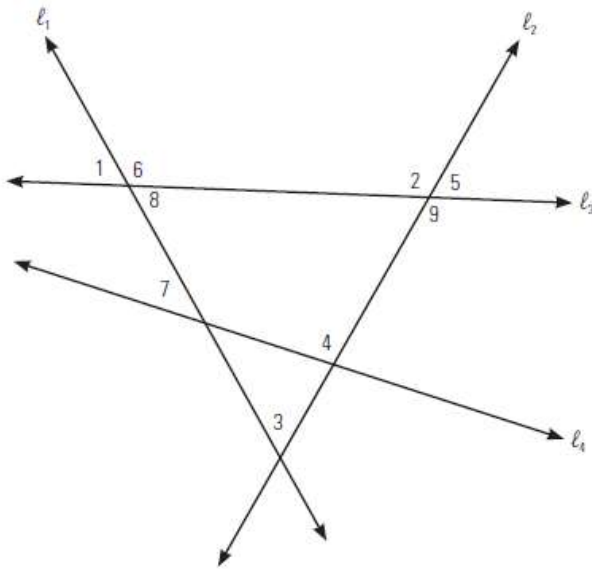
Corresponding Angles - Two angles that are congruent and occupy the same relative position in similar figures or at different intersections.

Vertically Opposite Angles - angles created by intersecting lines that share only a vertex.

Alternate Interior Angles - Angles in opposite positions between two lines intersected by a transversal and also on alternate sides of the same transversal.

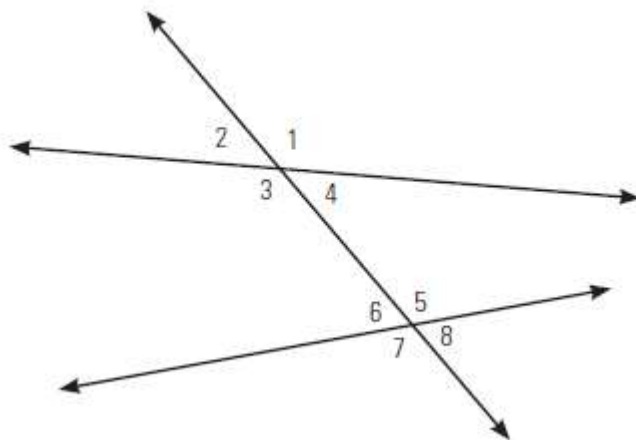
Alternate Exterior Angles - Angles in opposite positions outside two lines intersected by a transversal.

Example 1: In the following diagram, identify each of the following, and specify which lines and transversals you are using.



- an interior angle on the same side of the transversal as $\angle 6$
- an angle corresponding to $\angle 2$
- an angle corresponding to $\angle 4$
- an alternate interior angle to $\angle 4$

Example 2: In the diagram below, measure and record the sizes of the angles. Identify pairs of equal angles and state why they are equal.

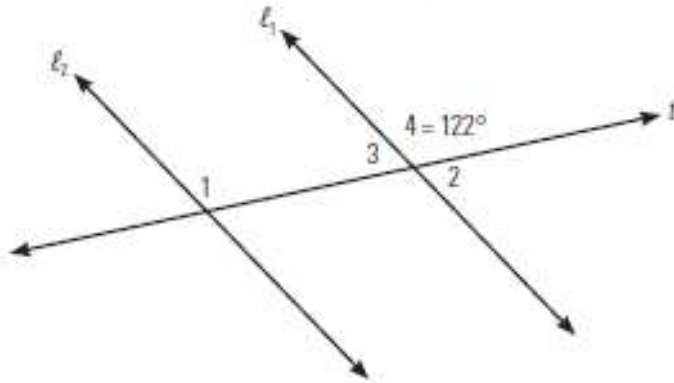


Lesson 5- Parallel Lines and Transversals

<p>If two parallel lines are intersected by a transversal:</p> <ul style="list-style-type: none">• The alternate interior angles are equal;• The corresponding angles are equal;• The same side interior angles are supplemental.	<p>If you know that, given two lines cut by a transversal:</p> <ul style="list-style-type: none">• Alternate interior angles are equal; or• Corresponding angles are equal; or• Same side interior angles are supplementary; <p>Then you can conclude that the lines are parallel.</p>
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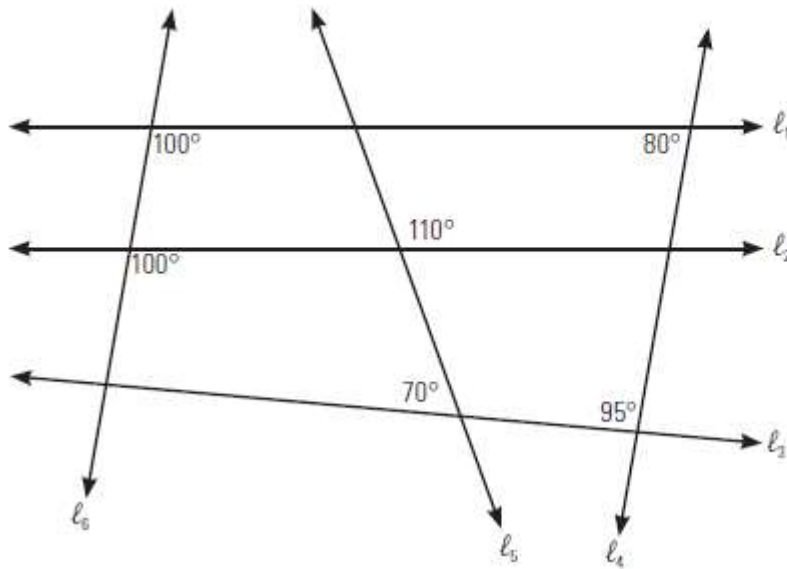
Example 1:

Consider the diagram below, in which ℓ_1 is parallel to ℓ_2 . What are the measures of the three indicated angles? Explain how you reached your answers.



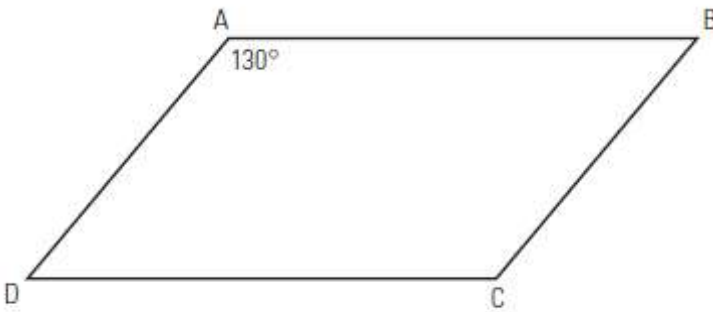
Example 2:

Given the diagram below, identify all the pairs of parallel lines and explain your selection.

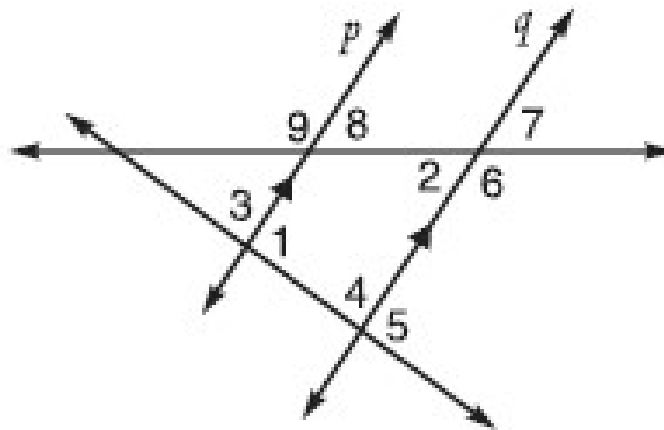


Example 3:

Given parallelogram ABCD, determine the values of $\angle B$, $\angle C$, and $\angle D$ in that order, stating your reason for each measure.

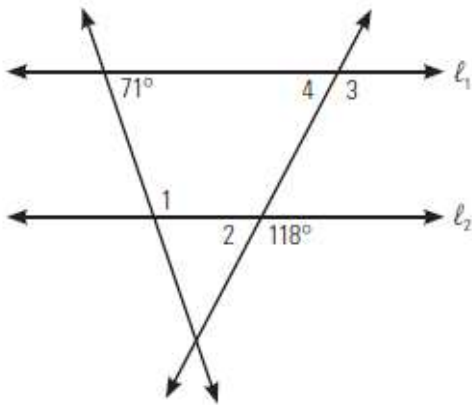


Example 4: In the figure below, lines p and q are parallel. Angle 1 is 78 degrees. Angle 2 is 47 degrees. Find the measure of each angle below.

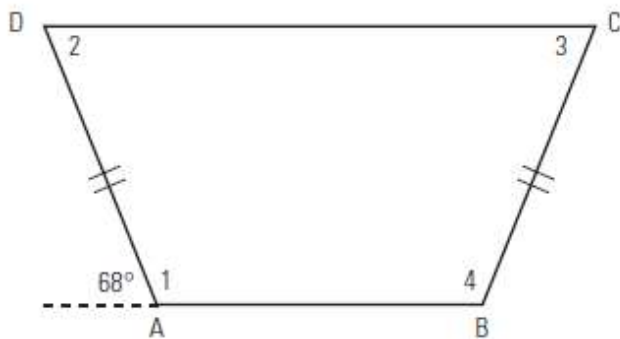


Worksheet 5 – Parallel Lines and Transversals

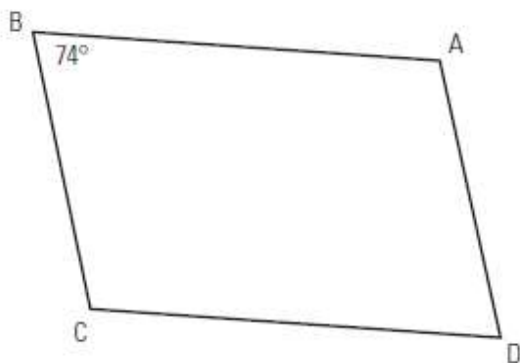
1. In the diagram below, l_1 is parallel to l_2 . State the measures of the indicated angles and explain your reasoning.



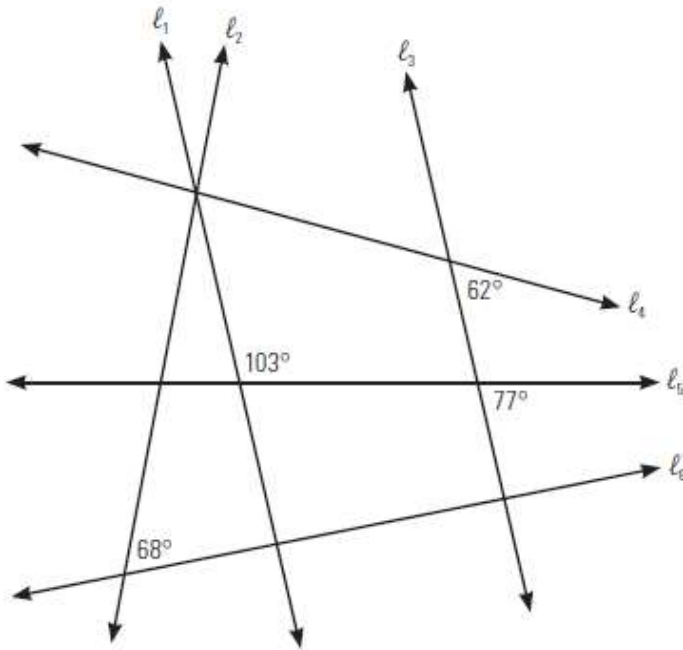
2. What are the measures of the interior angles in the trapezoid shown below? (Hint: Be careful of the order in which you calculate the angles.)



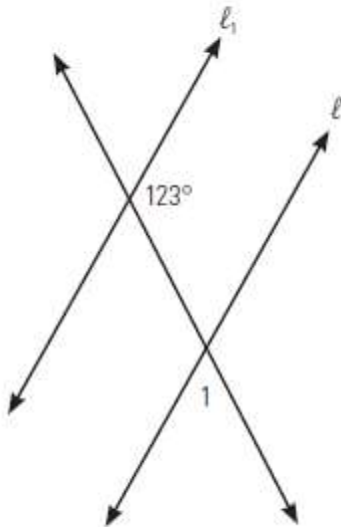
3. Quadrilateral ABCD is a parallelogram in which $\angle B$ measures 74° . Determine the measures of the other angles and state your reasons.



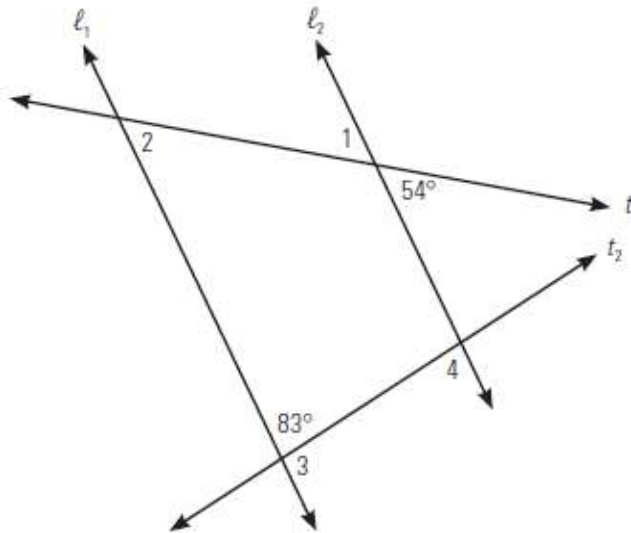
4. Find a pair of parallel lines in the following diagram. On the diagram, mark all the angles necessary to determine this.



5. What size must $\angle 1$ be if l_1 is parallel to l_2 ?



7. If l_1 and l_2 are parallel and are intersected by transversals t_1 and t_2 , what are the measures of the indicated angles? Solve for the measures in the given order, stating your reasoning.



SOLVING ANGLE MEASURES	
Angle Measure	Reason
$\angle 1 =$	
$\angle 2 =$	
$\angle 3 =$	
$\angle 4 =$	